ON THE EXACTNESS OF SPECIFICATION OF BOUNDARY RESISTANCES IN THE SOLUTION OF CERTAIN PROBLEMS OF NON-STEADY-STATE HEAT CONDUCTION WITH BOUNDARY CONDITIONS OF THE THIRD KIND BY THE NET METHOD

O. T. Il'chenko and V. E. Prokof'ev

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The present paper contains the results of an experimental study of the effect of the exactness of specification of boundary resistances on the temperature distribution in a steam turbine rotor in transient states as determined by means of a net model.

The exactness of solutions of heat conduction problems obtained by means of modeling devices depend not only on the calculation accuracy and on the choice of the net of resistances of the object; it is also dependent in large measure on the exactness of specification of the heat transfer conditions at the boundaries of the region under investigation.

In problems with boundary conditions of the third kind the resistances modeling heat transfer between the object and medium can be found from the following formulas, depending on the type of boundary point (see Fig. 1a):

for points obtained at the intersection of the net lines r with the boundary surface,

$$R_{b_m} = -\frac{2r}{2r \pm h_{4.5}} \times \frac{4\gamma h_{5.4} K_1}{(h_2 + h_3)(h_6 + h_7)\lambda(r \pm \gamma h_{5.4}/2,\ z,\ \varphi)} \times \frac{\lambda(r \pm \gamma h_{5.4}/2,\ z,\ \varphi)}{M(r_1 - r_m)\cos(\widehat{n, r})}\,\frac{1}{\alpha_b}; \tag{1}$$

for points obtained at the intersection of the net lines z with the boundary surface,

$$R_{b_k} = -\frac{4\beta h_3 K_1}{(h_4 + h_5)(h_6 + h_7)\lambda(r,\ z + \beta h_3/2,\ \varphi)} \times \frac{\lambda(r,\ z + \beta h_3/2,\ \varphi}{M(z_1 - z_k)\cos(\widehat{n, z})}\,\frac{1}{\alpha_b}; \tag{2}$$

for points obtained at the intersection of the net lines φ with the boundary surface,

$$R_{b_l} = -\frac{4\zeta h_6 K_1}{(h_2 + h_3)(h_4 + h_5)\lambda(r,\ z,\ \varphi - \zeta h_6/2)} \times \frac{\lambda(r,\ z,\ \varphi - \zeta h_6/2)}{(h_{\varphi_2} - h_{\varphi_l})\cos(\widehat{n, \varphi})M}\,\frac{1}{\alpha_b}; \tag{3}$$

for a nodal point at the surface,

$$R_{b_a} = -[\lambda(r,\ z,\ \varphi)K_1] \times \left[\frac{2r - h_5}{2r}\,\frac{(h_2 + h_3)(h_6 + h_7)}{4h_4} \times \lambda\left(r + \frac{h_4}{2},\ z,\ \varphi\right)(r_1 - r_a)\cos(\widehat{n, r}) + \frac{(h_4 + h_5)(h_6 + h_7)}{4h_3} \times \lambda\left(r,\ z + \frac{h_3}{2},\ \varphi\right)(z_1 - z_a)\cos(\widehat{n, z}) + \frac{(h_2 + h_3)(h_4 + h_5)}{4h_6}\lambda\left(r,\ z,\ \varphi - \frac{h_6}{2}\right) \times (h_{\varphi_1} - h_{\varphi_a})\cos(\widehat{n, \varphi})\right]^{-1}\frac{1}{M\alpha_b}. \tag{4}$$

Relations (1)–(4) were derived for a three-dimensional region in cylindrical coordinates by the method described in [1].

Since errors in measurement of the net branches affect both the interior and boundary net resistances, it is possible to incorporate this error in the scale factor K_1. The accuracy of determining the boundary resistances is then affected by the cosines between the exterior normals and net lines and by the criterion of heat transfer intensity α_b/λ.

If the boundaries of the domain under investigation are defined by a contour consisting of lines parallel and perpendicular to the net lines, the error in the quantities R_b due to errors of measuring of the angles between the exterior normals and net lines does not exceed 1.5–2.5%. It is clear that the accuracy of computed values of the boundary resistances is determined in this case largely by the accuracy of the data concerning the heat transfer conditions at the boundaries.

The question of exactness of specification of the boundary resistances is especially relevant in solving the heat conduction problem with time-dependent boundary conditions. This class of problems includes those of heating of steam turbine elements during start-up over sliding parameters. The fact that electrical modeling machines do not incorporate devices which would permit the specification of continuous time-dependent boundary conditions makes it necessary to use the Liebman method [2] in solving such problems. At the same time, our limited knowledge of the effect of exactness of specification of boundary conditions on the temperature distribution in the body makes it necessary to reduce the number of steps in the Liebman method with constant boundary resistances and to increase the number of respecification of these resistances.

We carried out our study with the purpose of determining the effect of heat transfer intensity on temperature distributions in a steam turbine rotor in simple heating problems and estimating permissible errors in the specification of boundary resistances in modelling. Since we varied R_b only by way of α_b, all our conclusions concerning the exactness of specification of boundary resistances apply to the effect of heat transfer intensity.

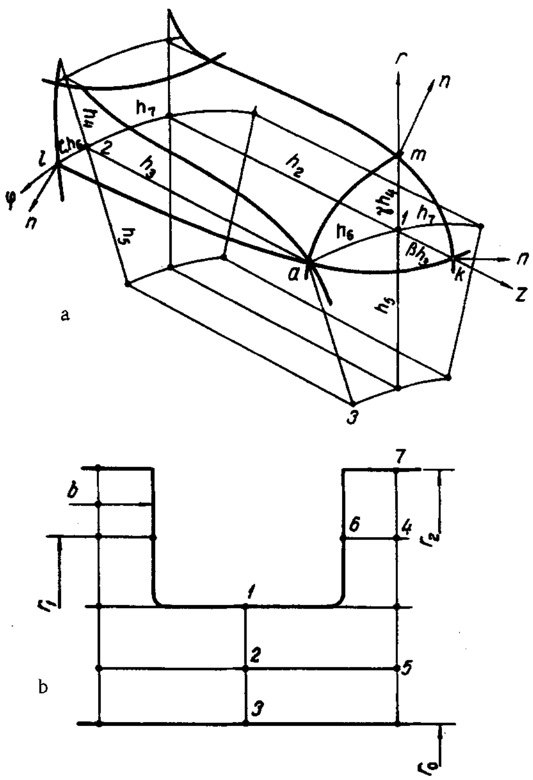

Fig. 1. a) Diagram of possible types of boundary points for a three-dimensional domain with a cylindrical net. b) Steam turbine rotor element.

The study was carried out on an MSM-1 modeling machine modernized to solve non-steady-state problems of heat conduction by the Liebman method [2]. In the course of it we solved several problems of simple heating of a steam turbine rotor over a wide range of variation of heat transfer coefficients at its boundaries. The measured results were plotted for the most characteristic points (Fig. 1b) of the rotor as functions of the relative temperature $\bar{\theta} = f(\text{Bi}, \tau)$.

Since the rotor of a multistage turbine consists largely of the elements shown in Fig. 1b, it follows that in estimating the effect of heat transfer intensity on the thermal state and exactness of specification of the boundary resistances we can limit ourselves to the analysis of the relations $\bar{\theta} = f(\text{Bi}, \tau)$ for the characteristic points of one such element.

The curves of Fig. 2 which represent the relative temperature $\bar{\theta} = f(\text{Bi}, \tau)$ for points lying on the surface of the rotor between the disks (the diaphragm condensation zone, point 1 in Fig. 1b) show that the requirements as regards the calculation accuracy and the exactness of the set of boundary resistances in solving heat conduction problems by the net method depend essentially on the heat transfer intensity and duration of the process. Deviations in Bi of ±10–15% for values Bi < 2.0 and ±20% for Bi > 2.0 do not affect the temperature distribution in the body of the rotor to within the accuracy of solution of problems by the net method. At the same time, we see that the requirements as to calculation accuracy and selection of boundary resistances for Bi > 6.0 are still less stringent.

For Bi > 15.0 and a process duration of over 15 min further increases in heat transfer intensity in determining the thermal state of the rotor have practically no effect on the temperature distribution at the various points of the region under investigation. This fact enables us to ignore these changes in investigating processes with time-variable boundary conditions of the third kind for Bi > 15.0.

Analysis of the curves of variation of the relative temperatures $\bar{\theta} = f(\text{Bi}, \tau)$ for points lying within the body of the rotor not only confirmed all that we have said concerning surface points; it also showed that for interior points far removed from the heat transfer surfaces with values Bi > 4.0, the duration of the process has no effect on the requirements as regards the exactness of specification of the boundary resistances.

The curves in Fig. 3, which represent the relative temperature $\bar{\theta} = f(\text{Bi}, \tau)$ for point 4 in Fig. 1b (i.e., within the body of the disk itself) indicate that the temperature distribution obtained in a thin element of a disk blade is satisfactory if the error of calculation and specification of boundary resistances under time-variable conditions of the third kind does not exceed ±10–15% for Bi < 4 and ±20% for Bi > 4. As in the case of points in the diaphragm condensation zone, the requirements as to calculation accuracy and selection of boundary resistances are much less stringent for Bi > 6–8.

The curves of Fig. 4, which represent the relative temperature for nodal points on the disk blades show that a deviation in Bi of ±10–15% for Bi < 0.65 and of ±20% for Bi > 0.65 do not affect the temperature distribution in the disk blades and in the rotor as a whole.

We note that the disk thickness was taken as the determining dimension in our estimates of heat transfer at the blades. Thus, equal heat transfer intensities at the blades and in the diaphragm condensations correspond to an approximately threefold difference in Bi.

The requirements as to calculation accuracy and boundary resistance specification for points on the disk blades are much less stringent for Bi > 0.65; for Bi > 3.0 and process durations in excess of 15 min further increases in heat transfer intensity have practically no effect on the temperature distribution in the region under investigation. It is clear that for points on the disk blades in problems with time-variable boundary conditions of the third kind these variations in the boundary resistances can be ignored upon attainment of values Bi > 3.0.

All we have said implies that the minimum permissible error in the determination of boundary resistances for any point on the rotor does not exceed ±10–15%. Increases in the permissible error are associated with increases in Bi. The results of our study

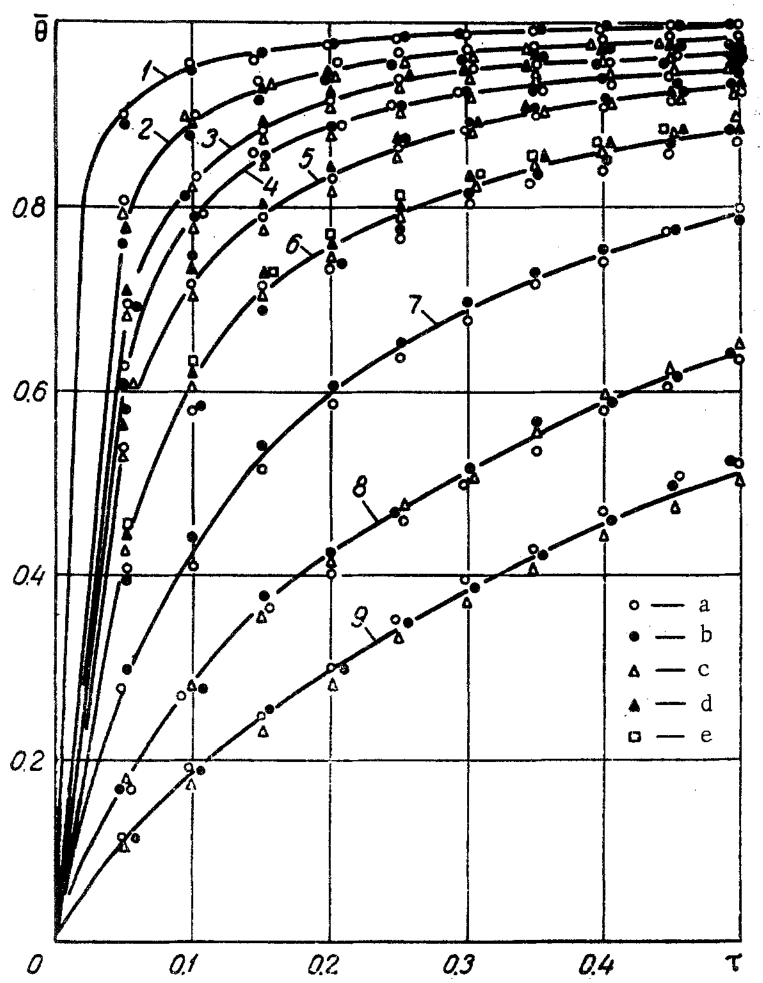

Fig. 2. Variation of the relative temperature $\bar{\theta} = f(\mathrm{Bi}, \tau)$ for points on the rotor surface in the diaphragm condensation zone (point 1 in Fig. 1b). 1) for Bi > 20 (a—45; b—37); 2) Bi = 14.0 (a—16.5; b—11.65; c—15.4; d—12.65); 3) 8.0 (a—7.59; b—9.24; c—7.9; d—10.0); 4) 6.0 (a—6.3; b—5.6; c—5.06); 5) 4.0 (a—4.2; b—3.8; c—3.50; d—3.60); 6) 2.0 (a—2.1; b—1.89; c—2.53; d—2.32; e—2.32); 7) 1.2 (a—1.17; b—1.27); 8) 0.55 (a—0.58; b—0.47; c—0.633); 9) 0.3 (a—0.29; b—0.33; c—0.25); τ is in hours.

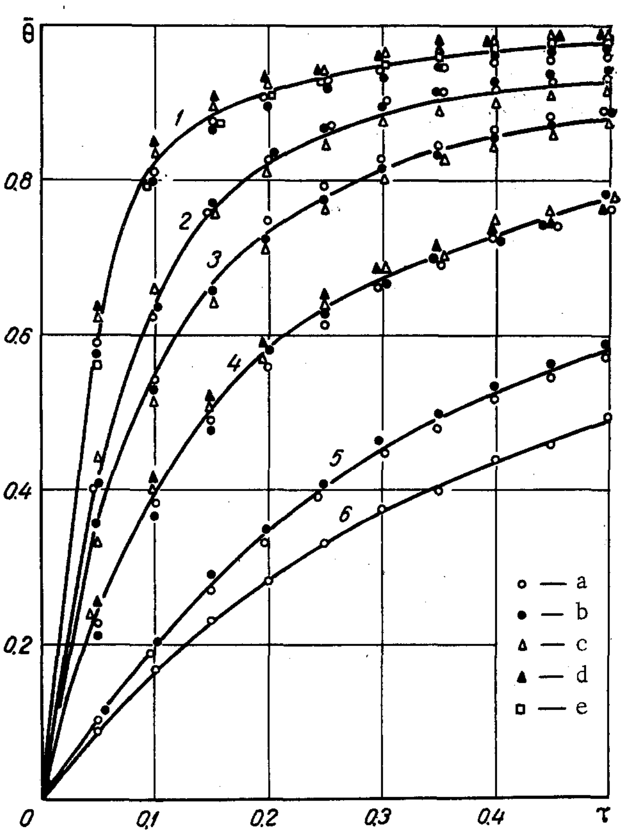

Fig. 3. Variation of the relative temperature $\bar{\theta} = f(\mathrm{Bi}, \tau)$ for points inside the disk itself (point 4, Fig. 1b). 1) for Bi > 18.0 (a—50; b—35; c—28; d—30; e—17); 2) Bi = 12.0 (a—12.7; b—11.5; c—13.7); 3) 6.0 (a—6.03; b—5.8; c—6.4); 4) 2.5 (a—3.01; b—2.90; c—2.8; d—2.3); 5) 1.5 (a—1.59; b—1.44); 6) 1.0.

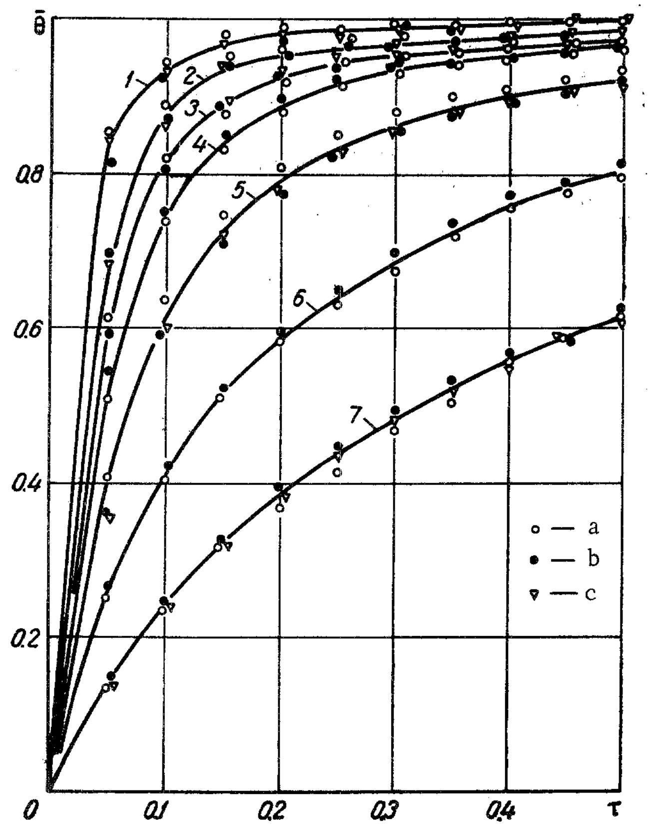

Fig. 4. Variation of the relative temperature $\bar{\theta} = f(\mathrm{Bi}, \tau)$ for points on the disk blades (point 6 in Fig. 1b). 1) Bi = 3.5 (a—3.8; b—3.5; c—3.44); 2) 2.0 (a—2.42; b—1.9; c—2.1); 3) 1.5 (a—1.4; b—1.55; c—1.5); 4) 0.65 (a—0.60; b—0.75); 5) 0.45 (a—0.49; b—0.47; c—0.37); 6) 0.2 (a—0.19; b—0.24); 7) 0.11 (a—0.135; b—0.12; c—0.094); τ is in hours.

showed that the level of Bi values permitting equal errors in the specification of R_b depends essentially on the nature of the point and varies over a very wide range. Hence, in studying the effect of heat transfer intensity on the permissible error of specification of boundary resistances it is advisable to use functions of the form $\bar{\theta} = f(\alpha/\lambda, \tau)$.

Conversion to such functions generalizes more fully the data contained in Figs. 2–4 from the standpoint of the permissible error in the specification of boundary resistance. In fact, a permissible error of the order ±10–15% in the specification of R_b corresponds to a ratio $\alpha_b/\lambda < 8.0$ for all of the types of points considered; a permissible error of ±20% corresponds to $\alpha_b/\lambda > 8.0$. For $\alpha_b/\lambda \geq 30.0$ further increases in the heat transfer intensity with time-variable boundary conditions of the third kind do not affect the accuracy and level of the temperatures to be determined in the solution of heat conduction problems by means of net models and need not be allowed for in the boundary resistances.

NOTATION

$\cos(n,\hat{}r)$, $\cos(n,\hat{}z)$, and $\cos(n,\hat{}\varphi)$ are the cosines between the exterior normal and the net lines at the boundary; h_2, h_3, h_4, h_5, h_6, and h_7 are the net spacings in the case of an irregularly spaced system; $\gamma h_{5,4}$, βh_3, and ξh_6 are incomplete spacings on the net; K_1 is the scale factor; M is the scale of the model; α_b is the heat transfer coefficient at the boundaries of the region; $\lambda(r, z, \varphi)$ is the thermal conductivity of the material as a function of the coordinates.

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Lenin Polytechnic Institute, Khar'kov